A Real-time Heart Rate Analysis for a Remote Millimeter Wave I-Q Sensor

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***Abstract*—This article analyzes Heart Rate (HR) information from physiological tracings collected with a remote millimeter wave (mmW) I-Q sensor for biometric monitoring applications. A parameter optimization method based on the nonlinear Levenberg-Marquardt (LM) algorithm is used. The mmW sensor works at 94 GHz and can detect the vital signs of a human subject from a few to tens of meters away. The reflected mmW signal is typically affected by respiration, body movement, background noise and electronic system noise. Processing of the mmW radar signal is thus necessary to obtain the true HR. The down-converted received signal in this case consists of both the real part (I-branch) and the imaginary part (Q-branch), which can be considered as the cosine and sine of the received phase of HR signal. Instead of fitting the converted phase angle signal, the method directly fits the real and imaginary parts of the HR signal, which circumvents the need for phase unwrapping. This is particularly useful when the signal-to-noise ratio (SNR) is low. Also the method identifies both beat-to-beat HR and individual heartbeat magnitude, which is valuable for some medical diagnosis applications. The mean HR here is compared to that obtained using the Discrete Fourier Transform (DFT).**

***Index Terms*— Heart Rate; Millimeter wave radar; Algorithm**

I. Introduction

HEART Rate (HR) analysis has been extensively investigated in the past decades [1]-[5]. The Low Frequency (LF) signal component (0.015-0.15 Hz) and the High Frequency (HF) signal component (0.15-0.4 Hz), are related with sympathetic and parasympathetic outflows, as well as respiratory rhythm; Also the LF/HF ratio reveals information about the balance between sympathetic and parasympathetic outflows [6], [7]. Conventional electrical or mechanical sensor, e.g., electrocardiogram (ECG), requires direct attachment to the subject [1]-[5]. Recently, there has been an increasing interest in remote contactless detection of HR using ultrasonic [8]-[10] or electromagnetic [11]-[15] sensors for biometrics applications. Such non-invasive electromagnetic sensors are particularly important under certain circumstances such as severe scald ambustions for which the use of contact sensors may not be viable [11]. In this article, we deal with a millimeter Wave (mmW) sensor working at 94 GHz, which was used to remotely measure HR at relatively long standoff distances.

Unlike the QRS complex of ECG signal, the received HR affected by complications introduced by respiration and body signal from contactless electromagnetic sensor is severe movement in addition to background noise and electronic system noise. So, the extraction of HR information generally requires employment of various signal processing methods, as has been done for QRS complex detection in ECG [16]-[23]. Due to the well-established QRS complex, different derivative methods and filter schemes have been successfully applied to ECG analysis [16]-[20]. However, direct application of such methods to HR signal detected with the remote electromagnetic sensor described in this article is not possible since the heartbeat complex is not well-defined as with QRS complex in ECG [11]-[15]. Recently, parameter optimization modeling has been proposed to analyze various biosignals [21]-[23]. In this article, we applied such parameter optimization method based on the nonlinear Levenberg-Marquardt (LM) algorithm [24]-[29] to extract the HR information by expressing the received I-branch (in-phase component) and Q-branch (quadrature-phase component) as the cosine and sine of the received signal, which can be expressed as the sum of heartbeat complex, respiration, body movement, background noise and electronic system noise. The advantages of our parameter optimization method compared to other methods like Discrete Fourier Transform (DFT) are: 1) the method requires no phase unwrapping—instead of fitting the converted HR signal, the fitting is done directly to the cosine (I-branch) and sine (Q-branch) of the received phase modulated signal, which is important when the SNR is low, and 2) the method obtains both beat-to-beat HR and individual heartbeat amplitude, critical for diagnosis of heart disease in certain applications.

II. 94-GHz Millimeter Wave Sensor

The schematic of the mmW system and the measurement setup is shown in Fig. 1. A W-band solid-state Gunn oscillator generates the reference and transmitted mmW signal which goes through a circulator, followed by a standard circular corrugated horn antenna and is focused by a 6-inch-diameter dielectric lens on the thorax area of a subject. The antenna system has a gain >25.0 dBi and a beam angle ~1°. The reflected mmW signal is then directly down-converted (homodyne) by an I-Q mixer. All data acquisition and processing are done by a personal computer using the LabVIEW™ software. The outputs (S_I , S_Q) for the I and Q

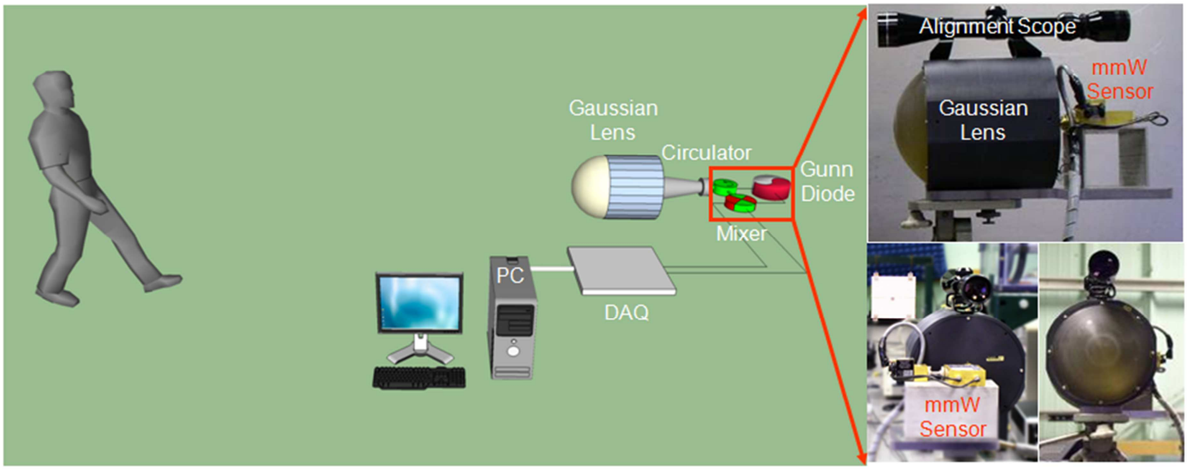

Fig. 1. Schematic of mmW transmitter-receiver system: the Gunn oscillator source generates the 94-GHz mmW signal, which is directed to a horn antenna via a circulator; the radiated mmW is focused on the target by a Gaussian lens; the reflected signal is then demodulated into I/Q-branch for baseband detection using a mixer and digitalized by a data acquisition (DAQ) board. All processing is done by LabVIEW™ running on a personal computer (PC).

branches containing the HR signal, i.e., the associated phase $\varphi(t)$, can be expressed as follows [11]-[15],

$$S_I = A_I(t)\cos[\varphi(t)] + n_I \quad (1)$$

$$S_Q = A_Q(t)\sin[\varphi(t)] + n_Q \quad (2)$$

where, $A_I(t)$, $A_Q(t)$ are the amplitudes determined by the gain setting for each branch; (n_I, n_Q) are added noise (including background noise and electronic system noise) for the I-branch and the Q-branch, respectively. For a calibrated quadrature mixer, $A_I(t) = A_Q(t)$

$$\varphi(t) = \arctan\left[\frac{S_Q - n_Q}{S_I - n_I}\right]$$

The HR signal or $\varphi(t)$ contains object distance $d(t)$ and Doppler frequency $f_d(t)$ information [15],

$$\varphi(t) = \frac{4\pi}{\lambda} d(t), \qquad f_d(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} \quad (3)$$

where λ is the operating wavelength of the sensor. We can express the signal $\varphi(t)$ as the sum of the heartbeat complex $H(t)$ and baseline composed of respiration and body movement $\xi(t)$,

$$\varphi(t) = \sum_{i=1}^{N}[a_i H(t-\tau_i)] + \xi(t) \quad (4)$$

where a_i is the amplitude of each heartbeat located at a center time of τ_i and N being the total number of heartbeats. For smooth respiration and body movement, we can expand the baseline signal in (4) as a Taylor series,

$$\xi(t) = \sum_{k=0}^{M}\frac{1}{k!}c_k t^k \quad (5)$$

where c_k is the Taylor coefficient and M being the order of expansion. Substituting Eq. (5) into Eq. (4) gives

$$\varphi(t) = \sum_{i=1}^{N}[a_i H(t-\tau_i)] + \sum_{k=0}^{M}\frac{1}{k!}c_k t^k \quad (6)$$

from which we can rewrite Eqs. (1) and (2) as

$$S_I = A_I(t)\cos\left[\sum_{i=1}^{N}[a_i H(t-\tau_i)] + \sum_{k=0}^{M}\frac{1}{k!}c_k t^k\right] + n_I \quad (7)$$

$$S_Q = A_Q(t)\sin\left[\sum_{i=1}^{N}[a_i H(t-\tau_i)] + \sum_{k=0}^{M}\frac{1}{k!}c_k t^k\right] + n_Q \quad (8)$$

III. Optimization Method for HR Analysis

The major task of HR analysis is to search for the optimized parameters in Eqs. (7) and (8), i.e., a_i, τ_i and c_k. The nonlinear LM algorithm is used for this purpose [24]-[29].

A. The LM algorithm

The LM algorithm [24-25] works by iteratively correcting the residue error through updating the parameter set $\vec{p}_j$,

$$\vec{p}_j = \vec{p}_{j-1} - \left(J^T J + \lambda diag\left[J^T J\right]\right)^{-1}\begin{bmatrix}\Delta S_I \\ \Delta S_Q\end{bmatrix} \quad (9)$$

where j denotes the j^{th} iteration; λ is the controlling parameter which is automatically adjusted during the each iteration; J is the Jacobian matrix consisting of columns of first derivatives of (S_I, S_Q) over the parameter set $\vec{p}$; $\Delta S_I = \overline{S_I - \tilde{S}_I}$ and $\Delta S_Q = \overline{S_Q - \tilde{S}_Q}$ are vector differences of the real and imaginary parts between actual measurement data (S_I, S_Q) and the fitted data ($\tilde{S}_I$, $\tilde{S}_Q$).

The numerical recipe for the LM algorithm is summarized in Fig. 2, which is also described below,

1) Specify the initial parameter set $\vec{p}_0$ and $\lambda = 0.01$. In our simulation, we adopt $a_0 = 1$ and $c_0 = c_1 = 0$ for first order approximation;

2) Calculate the parameter set and store it as a temporary vector $\vec{p}_{temporary}$ according to Eq. (9);

3) Calculate the χ^2 error of the current iteration of the j^{th}

iteration to see if it is smaller than the last, i.e., $(j-1)^{th}$ iteration,

$$\chi_j^{\,2} = \frac{1}{N}\left\{\sum_{j=1}^{N}\left[S_{I,j} - \tilde{S}_{I,j}\right]^2 + \sum_{j=1}^{N}\left[S_{Q,j} - \tilde{S}_{Q,j}\right]^2\right\} \tag{10}$$

with N being the number of sampling ensemble that contains the whole heartbeat.

4) If the $\chi_j^{\,2} < \chi_{j-1}^{\,2}$, then update the parameter set $\vec{p}_j = \vec{p}_{temporary}$ and set $\lambda_j = \lambda_{j-1}/10$; otherwise, maintain the old parameters and set $\lambda_j = 10\lambda_{j-1}$;

5) Repeat steps 2 to 4 until the χ^2 error becomes smaller than the specified value.

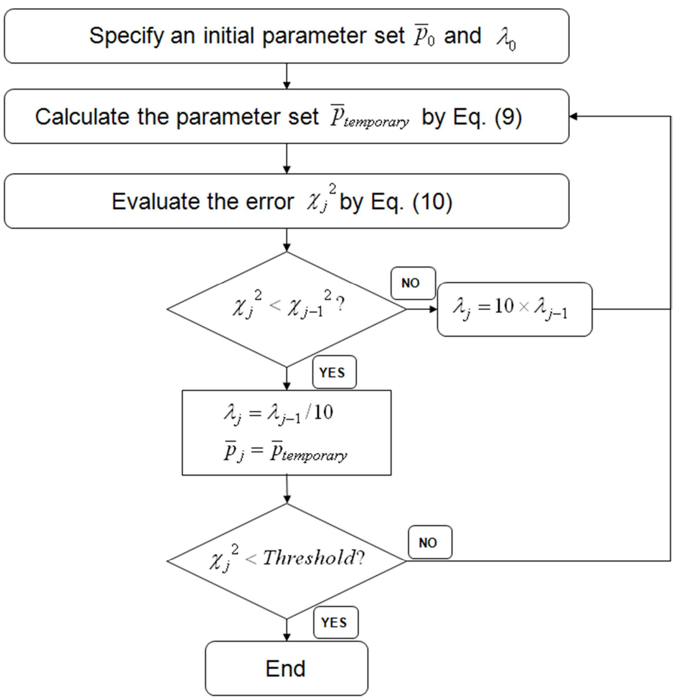

Fig. 2. Flow chart of nonlinear LM algorithm used for detection of HR.

B. The real-time optimization method

Although it is possible to fit to arbitrary duration of HR signal at the same time, in a real-time monitoring system, one usually fits a short-period (small segment) of HR signal containing only one heartbeat complex. For a calibrated quadrature mixer $A_I(t) = A_Q(t) = \sqrt{S_I^2 + S_Q^2}$. Below we summarize the procedure for applying the LM algorithm to single heartbeat complex for real-time HR analysis,

1) *Pick the reference heartbeat complex $H(t)$ from the trace where there is minimum influence from respiration and body movement;*
2) *Obtain a small segment of HR signal at time τ_i;*
3) *Run the nonlinear LM algorithm stated above (also in Fig. 2) and calculate the $\chi_i^{\,2}$ error of the i^{th} iteration;*
4) *Let $i = i+1$ and repeat steps 2) through 4) to obtain all optimized parameters where χ^2 is minimum.*

IV. Experimental Data and Results

Before we apply our method to real HR signal analysis, we first demonstrated that our mmW system works well by comparing it with the HR signal obtained by a commercially available laser vibrometer (Polytec® OFV-3000/OFV-302 helium-neon laser vibrometer). The signal show in Fig. 3 was collected from a simulated target made of a flat plate covered with a piece of rubber (mimics human skin). The signals were created by the sum of 3 arbitrary functions using a function generator: a sine wave for the baseline; a trapezoidal pulse to simulate respiration (0.5Hz respiration rate); and a right-half sinc pulse to simulate heartbeat (1.3Hz HR). From the comparison, one can clearly see that the mmW tracing closely follows that of the laser vibrometer. What's more important is that the mmW signal reveals more detail about the structure of heartbeat (the sinc function here).

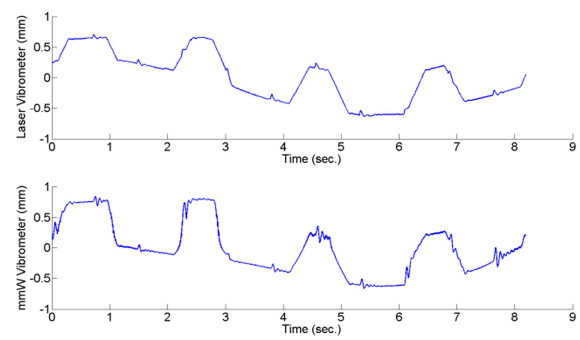

Fig. 3. Comparison between mmW (bottom) and laser (top) vibrometers using a simulated object consisting of a flat plate covered with a piece of rubber to mimic the human skin.

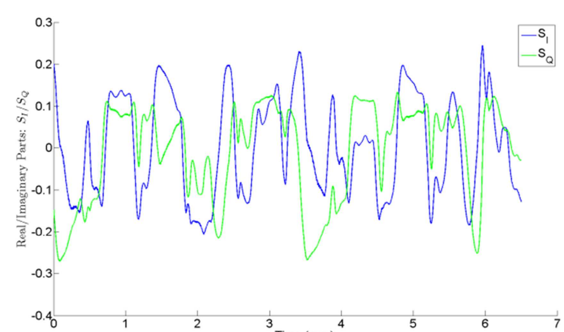

Fig. 4. Tracing A: real/imaginary parts (S_I , S_Q) of the raw signal taken from 5 meters away for a 94-GHz mmW vibrometer.

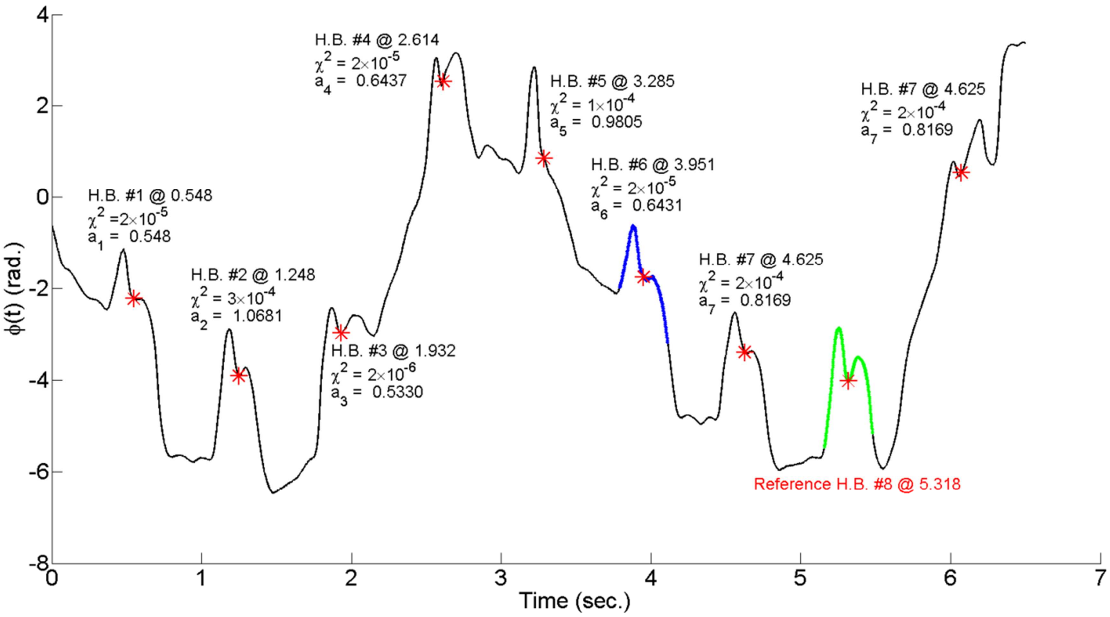

Fig. 5. Tracing A with respiration only: the optimization result of heartbeat #6 (the blue segment) are shown in Figs. 5 through 7. Heartbeat #8 (the green segment) is the reference heartbeat complex. The red asterisks are the centers of each heartbeat.

Now, let's analyze two types of HR tracings taken at a distance of 5m from the subject: A) HR tracing with respiration only; and B) HR tracing with both respiration and body movement. In each case, the HR is compared with that obtained using the Discrete Fourier Transform (DFT). All our data are taken with a 1-KHz sampling rate.

A. *HR signal with respiration only*

The tracing A was obtained when the subject was in front of the mmW sensor with arms stabilized to allow only small body movement. The real and imaginary parts of the data collected from a distance of 5 meters are shown in Fig. 4. The corresponding unwrapped phase $\varphi(t)$ is shown in Fig. 5, together with the parameter optimization result from the nonlinear LM algorithm described in Section III. The selected reference heartbeat complex $H(t)$ is heartbeat #8 (green dots). The center of each heartbeat is marked with a red asterisks.

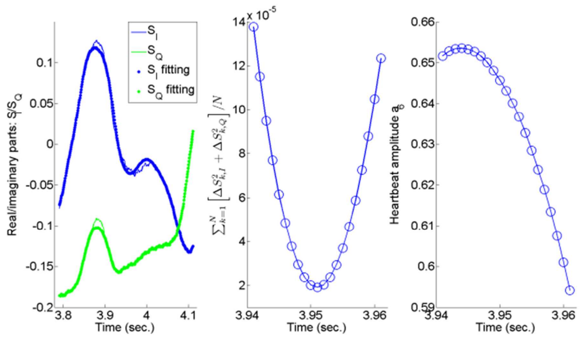

Fig. 6. Optimization result for heartbeat #6 of tracing A shown in Fig. 5: The left plot shows the agreements between measured real/imaginary parts with the fitting results; the middle plot shows the fitting error for different heartbeat center trials; and the right plot shows the fitting heartbeat amplitude for different heartbeat center trials.

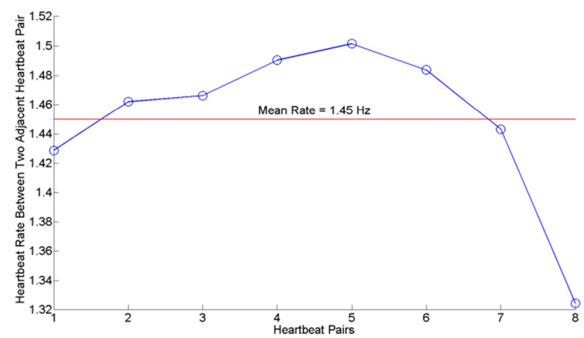

Fig. 7. Beat-to-beat HR for tracing A shown in Fig. 5.

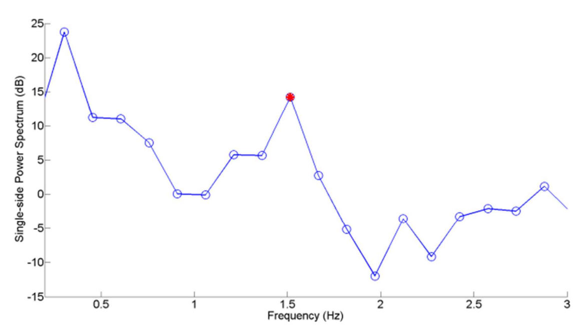

Fig. 8. DFT power spectrum of tracing A shown in Fig. 5. The red asterisk shows the peak HR=1.5 Hz.

The fitting error χ^2 and heartbeat amplitude a_i are also displayed for each heartbeat fit of the HR signal. The nonlinear LM fitting result is given in Fig. 6 for a typical heartbeat (heartbeat #6 in Fig. 5). The beat-to-beat HR is calculated from each heartbeat center in Fig. 5 and is plotted

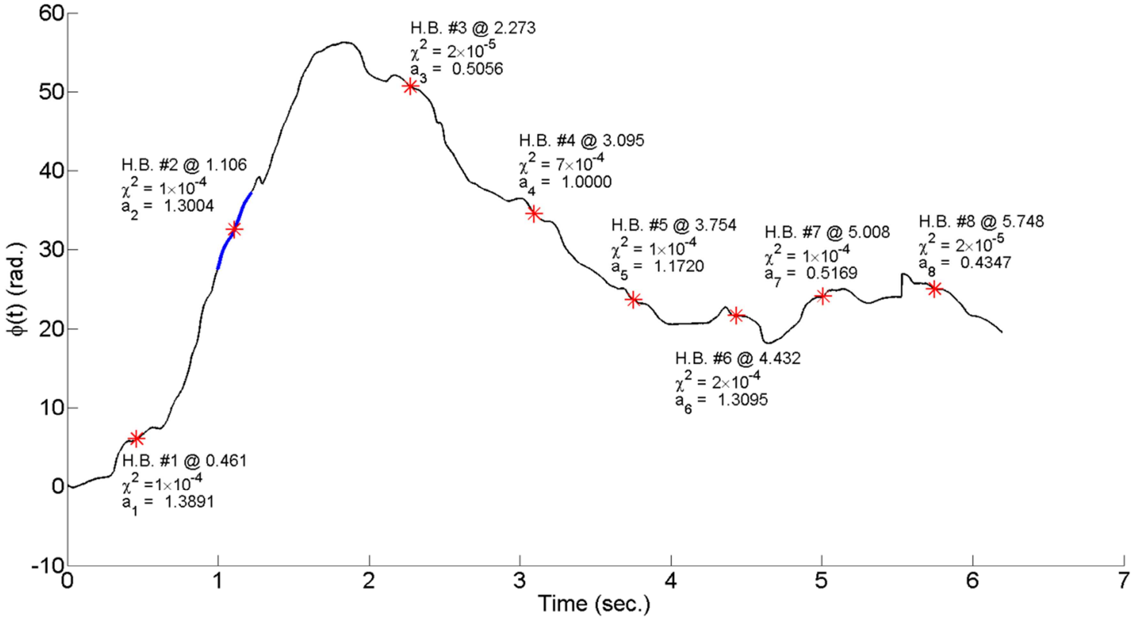

Fig. 9. HR tracing B with both respiration and body movement: the optimization result of heartbeat #2 (the blue segment) are shown in Fig. 9 to Fig. 11. The red asterisks are the centers of each heartbeat.

in Fig. 7. A mean value of HR=1.45 Hz is determined for that data. Also the mean magnitude is $\bar{a}$ = 0.7466 and the standard deviation is σ =0.2241, giving $(\sigma/\bar{a})^2$ = 0.0901, a quantity that indicates the normality of heartbeat activity: a low value means stable heartbeat activity. We also applied the DFT on the HR signal, the power spectrum of which is shown in Fig. 8. We obtained a peak HR=1.5 Hz (the red asterisk in Fig. 8), approximately the same as that obtained using the LM optimization method (HR=1.45 Hz above).

B. HR signal with both respiration and body movement

The tracing B was obtained with the mmW system without the subject being stabilized; i.e., small body movement was allowed. The mmW signal collected over approximately a 6s interval is shown in Fig. 9. The reference heartbeat complex $H(t)$ is the same as that of HR tracing A. The optimization result for this case is shown in Fig. 10. The method gives a mean value of HR=1.3862 Hz (Fig. 11). The mean magnitude is $\bar{a}$ = 0.9535 and the standard deviation is σ =0.4048, giving $(\sigma/\bar{a})^2$ = 0.1802. The DFT power spectrum of this tracing is shown in Fig. 12. Unlike HR tracing A, there is no obvious HR peak in the frequency spectrum.

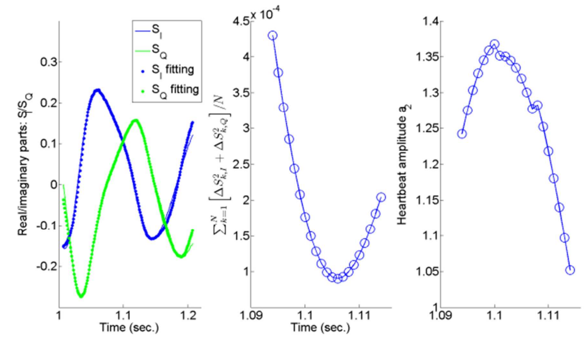

Fig. 10. Optimization result for heartbeat #2 of HR tracing B shown in Fig. 9.

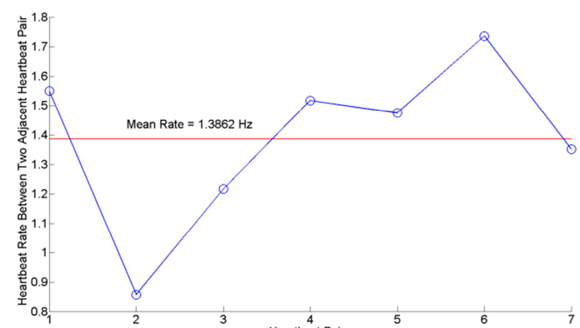

Fig. 11. Beat-to-beat HR for HR tracing B shown in Fig. 9.

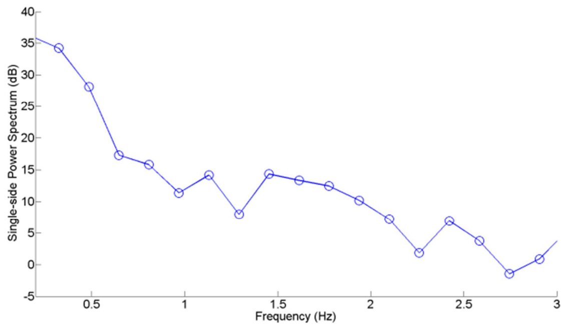

Fig. 12. DFT power spectrum of HR tracing B shown in Fig. 9. No obvious peak HR shows up for this case.

V. Discussion

We presented a parameter optimization method for HR analysis of a remote mmW I-Q sensor that shows robust performance for signals affected by respiration alone and by both respiration and body movement. The method can obtain both beat-to-beat HR and heartbeat amplitude information, even in presence of large respiration and body movement for which DFT analysis may not reveal any useful information about the mean HR. This is important for some critical applications such as for diagnosis of heart disease, sleep apnea, and for real-time monitoring of infants. For example, for sleep apnea application, beat-to-beat HR detection is critical since false alarm is common when the subject rolls over the bed. This is also true for seated baby monitoring because random movement can corrupt the HR signal easily. At last, our method could be combined with dual sensors random movement cancellation technique [30] to help improve their reliability, in which case, under-and-above or front-and-back sensors are used to simultaneously monitor the subject on a bed or in a seat.

Our mmW sensor provides a unique combination of sensitivity and long range for detection of biosignals [14]. Compared to microwave techniques [11]-[13], mmW range has the advantages of a) shorter wavelengths providing greater sensitivity to small displacements and b) higher spatial resolution obtainable with a reasonable aperture size [14]. Compared to laser Doppler technique, the main advantages of the mmW frequency range for remote sensing of biosignals include a) penetration through many optically opaque dielectric materials, b) low atmospheric attenuation allowing long-range operation, and c) low sensitivity to coarse reflecting surfaces, and d) ease of alignment [15]. Other possible techniques include the airborne ultrasound [8-10], which is low cost and can probe rough surface. However its lack of sensitivity, spatial resolution and narrow bandwidth limits its applications [31].

VI. Conclusion

We have presented a parameter optimization method based on the nonlinear LM algorithm for real-time HR analysis of data collected with a remote mmW I-Q sensor containing various sources of interfering signals including respiration, body movement and noise. The cases examined so far demonstrate that the algorithm is robust in that it is not sensitive to typical levels of noise. Two tracings of recorded data affected by respiration only and by both respiration and body movement have been analyzed. Good fitting results were obtained in both cases. The method fits the real and imaginary parts of the I-Q sensor, eliminating the need for phase unwrapping, which is particularly important when SNR is low. Additionally, the method provides both the beat-to-beat HR and the amplitude of each heartbeat, which is considered as critical information for some heart disease diagnosis applications.

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